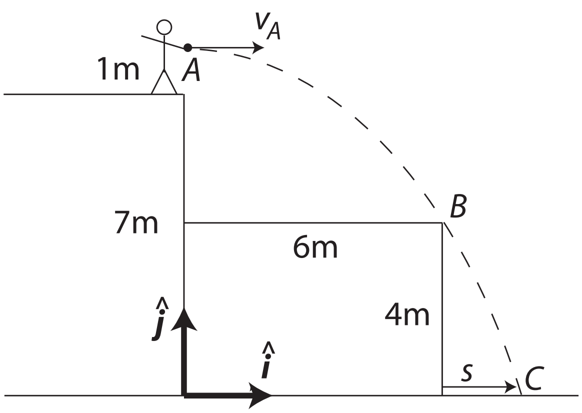
Example 2.2 (similar to M&K 5ed 2/63)
You are standing on top of the roof of a house. The house has a garage attached to the side of the house. You throw a baseball, releasing it from point A with a horizontal velocity. Find the necessary throwing speed so that the baseball just clears the corner of the garage, point B, and find the location of point C where the ball hits the ground.



We first define our coordinate system choosing the point on the ground where the garage joins the house as the origin. Next, we write the given information in vector form.

$$\boldsymbol{r}_A = 8\hat{\boldsymbol{j}} \tag{2.14}$$

$$\boldsymbol{v}_A = v_A\hat{\boldsymbol{i}} \tag{2.15}$$

$$\boldsymbol{a} = -9.81\hat{\boldsymbol{j}} \tag{2.16}$$

Let's say it takes t_B seconds for the ball to travel from A to B. At point B, the position vector is $\boldsymbol{r}_B = 6\hat{\boldsymbol{i}} + 4\hat{\boldsymbol{j}}$. We can substitute these vectors into the constant-acceleration equation.

$$\boldsymbol{r}_B = \boldsymbol{r}_A + \boldsymbol{v}_A t_B + \frac{1}{2}\boldsymbol{a}t_B^2 \tag{2.17}$$

$$6\hat{\boldsymbol{i}} + 4\hat{\boldsymbol{j}} = 8\hat{\boldsymbol{j}} + v_A t_B\hat{\boldsymbol{i}} - \frac{9.81}{2}t_B^2\hat{\boldsymbol{j}} \tag{2.18}$$

Next, we extract the scalar equations to get two equations for the two unknowns: v_A and t_B.

$$\begin{aligned} 6 &= v_A t_B \\ 4 &= 8 - \frac{9.81}{2}t_B^2 \end{aligned} \tag{2.19}$$

Solving for the unknowns: $t_B = 0.903$ s and $v_A = 6.64$ m/s.

To find the point of impact, let's say it takes t_C seconds for the ball to travel from A to C. At point C, the position vector is $\boldsymbol{r}_C = (6+s)\hat{\boldsymbol{\imath}}$.

$$\boldsymbol{r}_C = \boldsymbol{r}_A + \boldsymbol{v}_A t_C + \frac{1}{2}\boldsymbol{a} t_C^2 \tag{2.20}$$

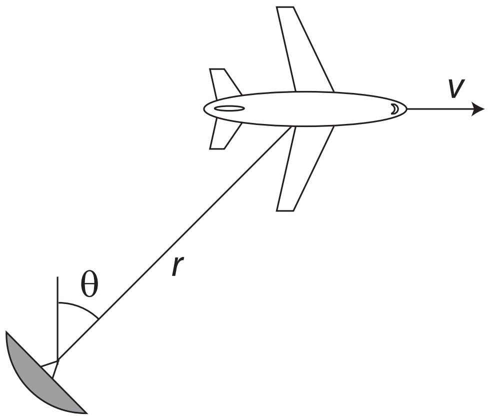
$$(6+s)\hat{\boldsymbol{\imath}} = 8\hat{\boldsymbol{\jmath}} + v_A t_C \hat{\boldsymbol{\imath}} - \frac{9.81}{2} t_C^2 \hat{\boldsymbol{\jmath}} \tag{2.21}$$

We can extract the scalar equations, using the solution for v_A that we just found.

$$\begin{aligned} 6+s &= 6.64 t_C \\ 0 &= 8 - \frac{9.81}{2} t_C^2 \end{aligned} \tag{2.22}$$

Solving for the unknowns: $t_C = 1.277$ s and $s = 2.49$ m.

Example 2.5
An airtraffic control radar is tracking an airplane flying eastward with constant velocity of 290 ft/sec. At the current instant, $\theta = 45°$ and $r = 30,000$ ft. Find $\dot{r}$, $\dot{\theta}$, $\ddot{r}$, and $\ddot{\theta}$.



Sketching our coordinate vectors, $\hat{\boldsymbol{e}}_r$ points from the radar toward the airplane, and $\hat{\boldsymbol{e}}_\theta$ is perpendicular to $\hat{\boldsymbol{e}}_r$ pointing down and to the right. We can use Eq. (2.34) to write the velocity vector.

$$\boldsymbol{v} = \dot{r}\hat{\boldsymbol{e}}_r + 30,000\,\dot{\theta}\hat{\boldsymbol{e}}_\theta \tag{2.45}$$

What other information do we have about the velocity vector? We know that the velocity is 290 ft/sec to the east. We can write this in vector format.

$$\boldsymbol{v} = 290\,(\sin 45°\,\hat{\boldsymbol{e}}_r + \cos 45°\,\hat{\boldsymbol{e}}_\theta) = 205.06\hat{\boldsymbol{e}}_r + 205.06\hat{\boldsymbol{e}}_\theta \text{ ft/sec} \tag{2.46}$$

From the previous two equations, we can now extract scalar equations.

$$\dot{r} = 205.06 \text{ ft/sec} \tag{2.47}$$

$$30,000\,\dot{\theta} = 205.06 \qquad \Rightarrow \qquad \dot{\theta} = 0.006835 \text{ rad/sec} \tag{2.48}$$

To find $\ddot{r}$ and $\ddot{\theta}$ we need to consider the acceleration of the airplane.

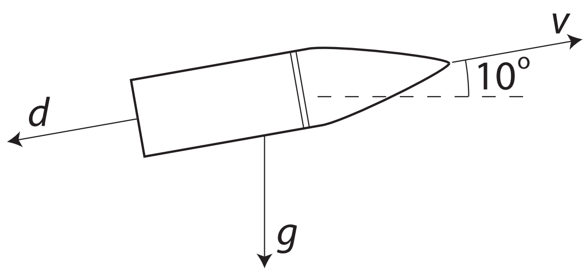
$$\boldsymbol{a} = \left(\ddot{r} - 30,000 \cdot 0.006835^2\right)\hat{\boldsymbol{e}}_r + \left(30,000\,\ddot{\theta} + 2 \cdot 205.06 \cdot 0.006835\right)\hat{\boldsymbol{e}}_\theta \tag{2.49}$$

From the given information that the velocity is constant, however, we know the acceleration is zero: $\boldsymbol{a} = 0\hat{\boldsymbol{e}}_r + 0\hat{\boldsymbol{e}}_\theta$. From this, we write two scalar equations.

$$\ddot{r} - 30,000 \cdot 0.006835^2 = 0 \qquad\qquad 30,000\,\ddot{\theta} + 2 \cdot 205.06 \cdot 0.006835 = 0 \tag{2.50}$$

Solving for the unknowns, $\ddot{r} = 1.40$ ft/sec^2 and $\ddot{\theta} = -9.34 \cdot 10^{-5}$ rad/sec^2.

Example 2.6
A bullet is fired with a velocity of 1000 m/s upward at an angle of 10°. The bullet's acceleration has two components: gravity $g = 9.81\text{m/s}^2$ and aerodynamic drag $d = 1160\ \text{m/s}^2$ which acts opposite the velocity. Find the radius of curvature of the bullet's trajectory and the rate at which the bullet's speed is changing.



Starting with sketching the coordinate vectors, $\hat{\boldsymbol{e}}_t$ is parallel to the velocity pointing to the right and slightly up, and $\hat{\boldsymbol{e}}_n$ points down and slightly to the right. Based on our kinematic expression, we can write the acceleration vector as the following.

$$\boldsymbol{a} = \dot{v}\hat{\boldsymbol{e}}_t + \frac{1000^2}{\rho}\hat{\boldsymbol{e}}_n \tag{2.55}$$

From the given information, we know that the acceleration vector is composed of two physical components. The drag component acts opposite the velocity vector, in the $-\hat{\boldsymbol{e}}_t$ direction. The gravity component acts straight down with components in both directions.

$$\boldsymbol{a} = -1160\hat{\boldsymbol{e}}_t + 9.81\left(\cos 10^\circ \hat{\boldsymbol{e}}_n - \sin 10^\circ \hat{\boldsymbol{e}}_t\right) \tag{2.56}$$

From these two expressions for the acceleration vector, we extract the scalar equations.

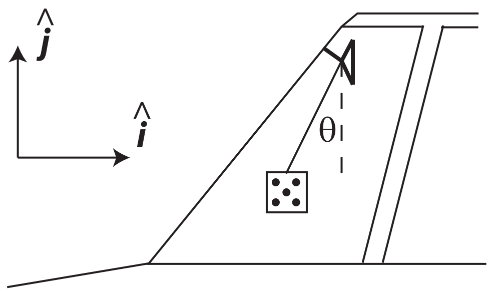
$$\dot{v} = -1160 - 9.81 \sin 10^\circ \tag{2.57}$$

$$\frac{1000^2}{\rho} = 9.81 \cos 10^\circ \tag{2.58}$$

Solving for the unknowns, $\dot{v} = -1162\ \text{m/s}^2$ and $\rho = 103,500\ \text{m} = 103.5\ \text{km}$.

3.3 Example Problems

Example 3.1
Consider a fuzzy die hanging on a cord from the rearview mirror of a car. The car is decelerating at $0.2g$. Find the steady-state angle the cord makes with the vertical.



1. Kinematics - Let's use the coordinate vectors indicated in the diagram. In the steady-state condition, the die's acceleration is the same as the car: $\boldsymbol{a} = 0.2g\hat{\boldsymbol{\imath}}$.
2. FBD - In sketching your own FBD, the only forces acting on the die are gravity, mg, and the tension in the cord, T. The resulting force expression is $\Sigma\boldsymbol{F} = T\sin\theta\hat{\boldsymbol{\imath}} + (T\cos\theta - mg)\hat{\boldsymbol{\jmath}}$.
3. N2L - Substituting into Newton's second law:

$$T\sin\theta\hat{\boldsymbol{\imath}} + (T\cos\theta - mg)\hat{\boldsymbol{\jmath}} = 0.2mg\hat{\boldsymbol{\imath}} \tag{3.5}$$

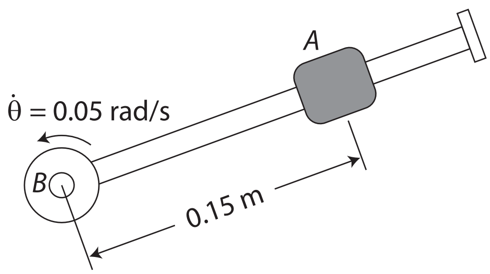
Extracting scalar equations of motion:

$$\begin{aligned} T\sin\theta &= 0.2mg \\ T\cos\theta - mg &= 0 \end{aligned} \tag{3.6}$$

From these equations, solve for θ.

$$\tan\theta = 0.2 \qquad \Rightarrow \qquad \theta = 11.31^\circ \tag{3.7}$$

Example 3.3
The rod rotates about B in the horizontal plane with a constant angular rate $\dot{\theta} = 0.05$ rad/s. Collar A has a mass of 0.3 kg and slides along the rod without friction. At the current instant, the collar is at a radius of $r = 0.15$ m and is sliding along the rod with $\dot{r} = 0.2$ m/s. Find the value of $\ddot{r}$ and the force exerted on the collar by the bar.



1. Kinematics - Choosing polar coordinate vectors, $\hat{\boldsymbol{e}}_r$ points along the rod, and $\hat{\boldsymbol{e}}_\theta$ points up and to the left. Using the given information, we can write an expression for the acceleration vector.

$$\begin{aligned} \boldsymbol{a} &= \left(\ddot{r} - 0.15 \cdot 0.05^2\right) \hat{\boldsymbol{e}}_r + (0 + 2 \cdot 0.2 \cdot 0.05)\, \hat{\boldsymbol{e}}_\theta \\ &= \left(\ddot{r} - 3.75 \cdot 10^{-4}\right) \hat{\boldsymbol{e}}_r + 0.02 \hat{\boldsymbol{e}}_\theta \end{aligned} \tag{3.13}$$

2. FBD - In sketching your own FBD, the rod exerts a force N on the collar. The force is perpendicular to the rod, but we're not sure if it's in the positive or negative $\hat{\boldsymbol{e}}_\theta$ direction. So for now, let's just guess it's in the positive $\hat{\boldsymbol{e}}_\theta$ direction.

$$\Sigma \boldsymbol{F} = N \hat{\boldsymbol{e}}_\theta \tag{3.14}$$

Of course, gravity is also acting on the collar. But since we're focused on the motion in the horizontal plane, we can neglect gravity.
3. N2L - Note that the collar has a mass of 0.3 kg.

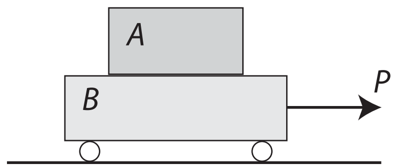
$$N \hat{\boldsymbol{e}}_\theta = 0.3 \left[\left(\ddot{r} - 3.75 \cdot 10^{-4}\right) \hat{\boldsymbol{e}}_r + 0.02 \hat{\boldsymbol{e}}_\theta\right] \tag{3.15}$$

Extracting scalar equations of motion:

$$\begin{aligned} 0 &= 0.3 \left(\ddot{r} - 3.75 \cdot 10^{-4}\right) \\ N &= 0.3 \cdot 0.02 \end{aligned} \tag{3.16}$$

From these equations, $\ddot{r} = 3.75 \cdot 10^{-4}$ m/s^2 and $N = 0.006$ N. We had guessed that N acts in the positive $\hat{\boldsymbol{e}}_\theta$ direction. Since we got a positive answer for N, this guess must have been correct.

Example 3.11
Block A has a mass of 50 kg, and block B has a mass of 10 kg. The coefficients of friction between blocks A and B are $\mu_s = 0.5$ and $\mu_k = 0.3$. Neglect any friction between block B and the ground. Find the acceleration of each block when a force of (a) $P = 200$ N and (b) $P = 300$ N is applied to block B.



In this problem, we need to analyze the motion of both blocks A and B. We'll start by assuming static friction between the blocks.
1. Kinematics - Rectangular coordinates with $\hat{\boldsymbol{\imath}}$ pointing to the right and $\hat{\boldsymbol{\jmath}}$ pointing up will be convenient for this problem. Under static friction, both A and B have the same acceleration, but the magnitude is unknown.

$$\boldsymbol{a}_A = \boldsymbol{a}_B = a\hat{\boldsymbol{\imath}} \tag{3.65}$$

Note, the static-friction assumption does not mean the blocks have zero acceleration!
2. FBD - The forces acting on A are its weight, a normal force N_1 from B, and the friction force f from B. Let's guess f pushes A to the right.

$$\Sigma\boldsymbol{F}_A = f\hat{\boldsymbol{\imath}} + (N_1 - 50g)\,\hat{\boldsymbol{\jmath}} \tag{3.66}$$

Two of the forces acting on B are its weight and a normal force N_2 from the ground. Also, the two forces N_1 and f that B exerts on A must be exerted in equal magnitude but opposite direction by A on B. And of course, there is the applied force P.

$$\Sigma\boldsymbol{F}_B = (P - f)\,\hat{\boldsymbol{\imath}} + (N_2 - N_1 - 10g)\,\hat{\boldsymbol{\jmath}} \tag{3.67}$$

3. N2L - We apply Newton's second law to each body.

$$\begin{aligned} A:&\quad f\hat{\boldsymbol{\imath}} + (N_1 - 50g)\,\hat{\boldsymbol{\jmath}} = 50a\hat{\boldsymbol{\imath}} \\ B:&\quad (P - f)\,\hat{\boldsymbol{\imath}} + (N_2 - N_1 - 10g)\,\hat{\boldsymbol{\jmath}} = 10a\hat{\boldsymbol{\imath}} \end{aligned} \tag{3.68}$$

From these two vector equations, we can extract four scalar equations of motion.

$$\begin{aligned} f &= 50a \\ N_1 - 50g &= 0 \\ P - f &= 10a \\ N_2 - N_1 - 10g &= 0 \end{aligned} \tag{3.69}$$

Solving for the unknowns, $a = P/60$, $f = 5P/6$, $N_1 = 490.5$ N, and $N_2 = 588.6$ N. From this solution, the maximum possible static friction force is $f_{\max} =$

$\mu_s N_1 = 245.25$ N. For $P = 200$ N, the required friction force is only $f = 166.67$ N, and the acceleration is $a = 3.33$ m/s^2. Comparing f to $f_{\max}$, the static assumption was good; so we're done.

For $P = 300$ N, however, the required friction force is $f = 250$ N, which is greater than the maximum possible friction force. Block A will actually be sliding to the left relative to block B in this case. We have to rework the problem using this realization.

1. Kinematics - Blocks A and B will actually have two different accelerations.

$$\boldsymbol{a}_A = a_A \hat{\boldsymbol{\imath}} \qquad\qquad \boldsymbol{a}_B = a_B \hat{\boldsymbol{\imath}} \tag{3.70}$$

2. FBD - Because block A is sliding to the left relative to block B, the kinetic friction force $f = \mu_k N_1$ is applied acting to the right on block A (and acting to the left on block B).

$$\begin{aligned} \Sigma \boldsymbol{F}_A &= \mu_k N_1 \hat{\boldsymbol{\imath}} + (N_1 - 50g)\,\hat{\boldsymbol{\jmath}} \\ \Sigma \boldsymbol{F}_B &= (P - \mu_k N_1)\,\hat{\boldsymbol{\imath}} + (N_2 - N_1 - 10g)\,\hat{\boldsymbol{\jmath}} \end{aligned} \tag{3.71}$$

3. N2L - We apply Newton's second law to each body.

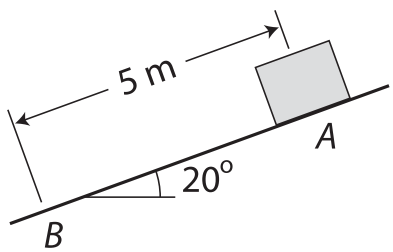
$$\begin{aligned} A:&\quad \mu_k N_1 \hat{\boldsymbol{\imath}} + (N_1 - 50g)\,\hat{\boldsymbol{\jmath}} = 50 a_A \hat{\boldsymbol{\imath}} \\ B:&\quad (P - \mu_k N_1)\,\hat{\boldsymbol{\imath}} + (N_2 - N_1 - 10g)\,\hat{\boldsymbol{\jmath}} = 10 a_B \hat{\boldsymbol{\imath}} \end{aligned} \tag{3.72}$$

From these two vector equations, we can extract four scalar equations of motion.

$$\begin{aligned} \mu_k N_1 &= 50 a_A \\ N_1 - 50g &= 0 \\ P - \mu_k N_1 &= 10 a_B \\ N_2 - N_1 - 10g &= 0 \end{aligned} \tag{3.73}$$

The case of $P = 300$ N gives the following solutions, $N_1 = 490.5$ N, $N_2 = 588.6$ N, $a_A = 2.943$ m/s^2, and $a_B = 15.285$ m/s^2.

Example 6.2
The 30 kg block slides 5 m down the ramp from A to B. The coefficient of kinetic friction between the block and the ramp is 0.3. The speed of the block at A is 4 m/s down the ramp. Find the speed when the block passes B.



We'll take a look at working this problem using both forms of the work-energy principle. First, the initial kinetic energy of the crate is $T_A = \frac{1}{2}30 \cdot 4^2 = 240$ J. Next, we need to consider the work done on the crate as it travels the ten meters from A to B. The forces acting on the crate are the normal and friction forces from the chute and weight due to gravity. The normal force acts perpendicular to the motion of the crate, so it does zero work. The friction force acts opposite the motion, so it does negative work. The friction force is related to the normal force, so we still need to use Newton's second law to find the normal force. The gravitational force has components parallel and perpendicular to the chute, but only the component parallel to the chute does work.

$$\begin{aligned} U_{A\to B} &= -5f + 5mg\sin 20^\circ = -5\mu_k N + 5mg\sin 20^\circ \\ &= -5\mu_k mg\cos 20^\circ + 5mg\sin 20^\circ = 88.455 \text{ J} \end{aligned} \tag{6.10}$$

The kinetic energy at B is $T_B = \frac{1}{2}30v_B^2$. Applying the work-energy principle:

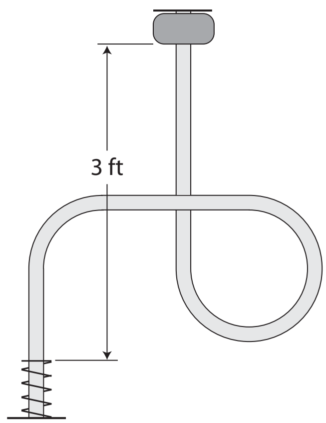
$$\begin{aligned} T_A + U_{A\to B} &= T_B \\ 240 + 88.455 &= \frac{1}{2}30v_B^2 \\ v_B &= 4.68 \text{ m/s} \end{aligned} \tag{6.11}$$

Now, let's work this problem using the potential-energy form of the work-energy principle. In writing the potential energy, let's define B as having a height of zero. Therefore, the potential energy at A is $V_A = mgh_A = 30 \cdot 9.81 \cdot 5\sin 20^\circ = 503.3$ J, and the potential energy at B is $V_B = mgh_B = 0$. The friction is the only nonpotential force that does work: $U_{NP,A\to B} = -5f = -5\mu_k N = -5\mu_k mg\cos 20^\circ = -414.8$ J.

$$\begin{aligned} T_A + V_A + U_{NP,A\to B} &= T_B + V_B \\ 240 + 503.3 - 414.8 &= \frac{1}{2}30v_B^2 \\ v_B &= 4.68 \text{ m/s} \end{aligned} \tag{6.12}$$

Whether we think about the $mg \sin 20°$ component of weight doing work over the 5 m distance, or think about the weight mg causing a change in potential energy due to the $5 \sin 20°$ height change, it's two sides of the same coin.

Example 6.3
A 5 lb slider is released from rest at the top of the smooth rod. Find the maximum compression of the 20 lb/in spring.



The slider is released from rest at the initial point, and therefore has zero kinetic energy: $T_1 = 0$. Defining the height of the spring in its undeformed configuration as the reference height, the slider has a potential energy of $V_1 = mgh = 5 \cdot 3 = 15$ ft $\cdot$ lb.

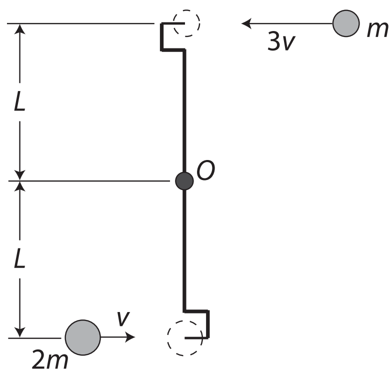
Since the rod is smooth, the only nonpotential force acting on the slider is the normal force from the rod, but it does no work: $U_{NP} = 0$. Because no nonpotential forces do work, it really doesn't matter what happens to the slider between the point of release and the point of maximum compression.

The maximum compression of the spring will come, at the instant when the slider will again have zero kinetic energy: $T_2 = 0$. At this instant there will be elastic potential energy in the spring, and the slider will have negative gravitational potential energy, since the compression in the spring will actually drop the slider below the reference height: $V_2 = \frac{1}{2}kx^2 - mgx = \frac{1}{2}20 \cdot 12x^2 - 5x$, where x is the spring compression in feet. Applying the work-energy principle:

$$\begin{aligned} T_1 + V_1 + U_{NP} &= T_2 + V_2 \qquad (6.13) \\ 15 &= 120x^2 - 5x \\ x &= \frac{9}{24} \text{ or } -\frac{1}{3} \text{ ft} = 4.5 \text{ or } -4 \text{ in} \end{aligned}$$

The quadratic equation for x gives two answers: one positive and one negative. The positive answer corresponds with compression of the spring. The negative answer corresponds represents if the collar had bounced back up the rod with the spring attached, stretching it. The work-energy principle can only tell us that these two solutions both have the correct amount of energy. It's up to us to recognize that the collar actually compresses the spring with the answer $x = 4.5$ in.

Example 6.9 (M&K 6ed 3/230)
The small spheres, which have the masses and initial velocities shown in the figure, strike and become attached to the spiked ends of the rod, which is freely pivoted at fixed point O and is initially at rest. Determine the angular velocity ω of the assembly after impact. Neglect the mass of the rod.



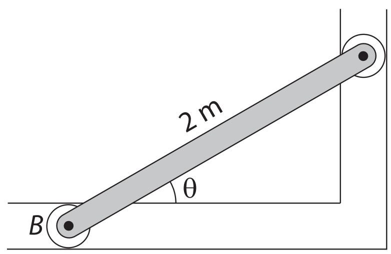
Define time 1 before the spheres impact the rod, and time 2 after the entire assembly is rotating about O. Define $\hat{\boldsymbol{k}}$ pointing up from the table shown in the diagram. At time 1, the $2m$ sphere has an angular momentum about O of $2mvL\hat{\boldsymbol{k}}$, and the m sphere has an angular momentum about O of $3mvL\hat{\boldsymbol{k}}$.

During the collision, forces act between each sphere and the spikes, and the rod may also experience some reaction forces at O. If we consider the system consisting of the two spheres and the rod, then the interaction forces between the spheres and the spikes impart zero net angular impulse on the system. The reaction forces at O generate zero moments about O, and therefore also impart zero net angular impulse about O. Once the system starts rotating about O, there must be some tension in the rod pulling the spheres through their new circular trajectories. But these forces also generate zero moment about O. Therefore, from time 1 to time 2, there is zero net angular impulse on the system.

At time 2, both spheres have a speed of ωL. The angular momentum about O of the $2m$ sphere is now $2m(\omega L)L\hat{\boldsymbol{k}}$, and the angular momentum about O of the m sphere is $m(\omega L)L\hat{\boldsymbol{k}}$. Assume the rod has negligible mass so we don't have to worry about its angular momentum. Substitute all of the above information into the angular momentum-impulse principle.

$$\boldsymbol{h}_{O,1} = \boldsymbol{h}_{O,2} \qquad \Rightarrow \qquad \begin{aligned} 2mvL\hat{\boldsymbol{k}} + 3mvL\hat{\boldsymbol{k}} &= 2m(\omega L)L\hat{\boldsymbol{k}} + m(\omega L)L\hat{\boldsymbol{k}} \\ 5mvL &= 3m\omega L^2 \\ \omega &= \frac{5v}{3L} \end{aligned} \tag{6.43}$$

Example 4.3
The end B of the bar moves in a horizontal channel to the right at a constant speed of 3 m/s. The end A of the bar moves in a vertical channel. At the instant when $\theta = 30°$, find the angular velocity and angular acceleration of the bar. Also, find the velocity and acceleration of A.



A. Define $\hat{\boldsymbol{\imath}}$ pointing to the right, $\hat{\boldsymbol{\jmath}}$ pointing up, and $\hat{\boldsymbol{k}}$ pointing out of the page.
B. The magnitude of the angular velocity is unknown, but it's clear it will be in the positive $\hat{\boldsymbol{k}}$ direction: $\boldsymbol{\omega} = \omega\hat{\boldsymbol{k}}$. The magnitude of the angular acceleration is unknown, and it's not even clear what the direction will be. Let's guess $\boldsymbol{\alpha} = \alpha\hat{\boldsymbol{k}}$.
C. Let's use B as a reference point: $\boldsymbol{v}_B = 3\hat{\boldsymbol{\imath}}$ and $\boldsymbol{a}_B = \boldsymbol{0}$. We also need the position of A relative to B.

$$\boldsymbol{r}_{A/B} = 2\left(\cos 30°\hat{\boldsymbol{\imath}} + \sin 30°\hat{\boldsymbol{\jmath}}\right) = \sqrt{3}\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}} \tag{4.15}$$

D. From this we can calculate the velocity and acceleration of A.

$$\begin{aligned} \boldsymbol{v}_A &= \boldsymbol{v}_B + \boldsymbol{\omega} \times \boldsymbol{r}_{A/B} \\ &= 3\hat{\boldsymbol{\imath}} + \omega\hat{\boldsymbol{k}} \times \left(\sqrt{3}\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}}\right) \\ &= (3 - \omega)\,\hat{\boldsymbol{\imath}} + \sqrt{3}\omega\hat{\boldsymbol{\jmath}} \end{aligned} \tag{4.16}$$

$$\begin{aligned} \boldsymbol{a}_A &= \boldsymbol{a}_B + \boldsymbol{\alpha} \times \boldsymbol{r}_{A/B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}_{A/B}) \\ &= \alpha\hat{\boldsymbol{k}} \times \left(\sqrt{3}\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}}\right) + \omega\hat{\boldsymbol{k}} \times \left(-\omega\hat{\boldsymbol{\imath}} + \sqrt{3}\omega\hat{\boldsymbol{\jmath}}\right) \\ &= -\left(\alpha + \sqrt{3}\omega^2\right)\hat{\boldsymbol{\imath}} + \left(\sqrt{3}\alpha - \omega^2\right)\hat{\boldsymbol{\jmath}} \end{aligned} \tag{4.17}$$

Physically, we know the velocity and acceleration of A are in the vertical direction: $\boldsymbol{v}_A = v_A\hat{\boldsymbol{\jmath}}$ and $\boldsymbol{a}_A = a_A\hat{\boldsymbol{\jmath}}$. From these two vector equations, we get four scalar equations.

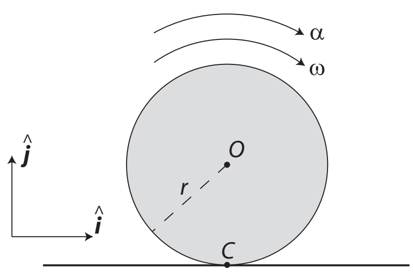
$$\begin{aligned} 0 &= 3 - \omega \\ v_A &= \sqrt{3}\omega \\ 0 &= \alpha + \sqrt{3}\omega^2 \\ a_A &= \sqrt{3}\alpha - \omega^2 \end{aligned} \tag{4.18}$$

Solving for the unknowns: $\omega = 3$ rad/s, $\alpha = -9\sqrt{3}$ rad/s^2, $v_A = 3\sqrt{3}$ m/s, $a_A = -36$ m/s^2.

4.4 Rolling Wheels

Rolling wheels are one type of rigid-body kinematics problem.

Example 4.6
Consider a wheel that is rolling without slipping. This means that the point of contact (the part of the wheel that is touching the ground) has the same velocity as the ground, i.e. zero. It doesn't mean that the point of contact has zero acceleration. If we come back an instant later, that same piece of the wheel will no longer be the point of contact and will have nonzero velocity.



Let's find the velocity of O, the center of the wheel.
A. The $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$ directions are shown in the diagram. To keep a right-handed coordinate system, $\hat{\boldsymbol{k}}$ points out of the page.
B. Based on the indicated directions, $\boldsymbol{\omega} = -\omega\hat{\boldsymbol{k}}$ and $\boldsymbol{\alpha} = -\alpha\hat{\boldsymbol{k}}$.
C. We can use C as a reference point, since we know its velocity: $\boldsymbol{v}_C = \mathbf{0}$. The position of O relative to C is $\boldsymbol{r}_{O/C} = r\hat{\boldsymbol{\jmath}}$.
D. From this we can calculate the velocity of O.

$$\boldsymbol{v}_O = \boldsymbol{v}_C + \boldsymbol{\omega} \times \boldsymbol{r}_{O/C} = -\omega\hat{\boldsymbol{k}} \times \hat{\boldsymbol{\jmath}} = \omega r\hat{\boldsymbol{\imath}} \tag{4.29}$$

Using calculus, we can find the acceleration of O.

$$\boldsymbol{a}_O = \frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{v}_O) = \alpha r\hat{\boldsymbol{\imath}} \tag{4.30}$$

For good measure, let's find the acceleration of C. Steps A and B don't change.
C. Since we found the acceleration of O, we can now use it as a reference point. The position of C relative to O is $\boldsymbol{r}_{C/O} = -r\hat{\boldsymbol{\jmath}}$.
D. Calculate the acceleration of C.

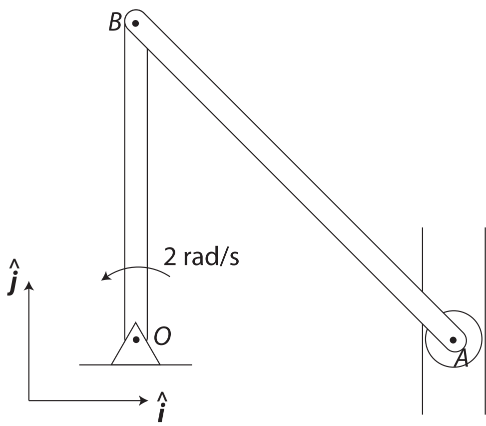
$$\begin{aligned}\boldsymbol{a}_C &= \boldsymbol{a}_O + \boldsymbol{\alpha} \times \boldsymbol{r}_{C/O} + \boldsymbol{\omega} \times \left(\boldsymbol{\omega} \times \boldsymbol{r}_{C/O}\right) \\ &= \alpha r\hat{\boldsymbol{\imath}} + \left(-\alpha\hat{\boldsymbol{k}}\right) \times \left(-r\hat{\boldsymbol{\jmath}}\right) + \left(-\omega\hat{\boldsymbol{k}}\right) \times \left[\left(-\omega\hat{\boldsymbol{k}}\right) \times \left(-r\hat{\boldsymbol{\jmath}}\right)\right] \\ &= \omega^2 r\hat{\boldsymbol{\jmath}}\end{aligned} \tag{4.31}$$

Thus the point of contact is accelerating up off the ground. Now that the we've found the velocity and acceleration of the center and point of contact of a wheel rolling without slipping, we can use these results in other problems.

4.5 Mechanisms

Mechanisms made up of multiple rigid bodies are another type of rigid-body kinematics problem. Each body has its own angular velocity and angular acceleration. We'll apply the same steps to these problems, just working from one end of the mechanism to the other. In doing this, we take advantage of the fact that a joint is a point on both of the bodies that it connects.

Example 4.9 (Similar to M&K 5ed 5/128)
Consider the mechanism made of links OB and BA. Link OB has length 1 m, and link AB has length $\sqrt{2}$ m. Link OB is rotating counter clockwise with a constant angular velocity of 2 rad/s. Point A rides in a vertical channel and at the current instant is 1 m to the right of point O. We want to find the angular velocity and angular acceleration of link AB.



A. The coordinate vectors are shown in the diagram.
B. The angular velocity and angular acceleration of OB are known, but the angular velocity and angular acceleration of AB are unknown.

$$\boldsymbol{\omega}_{OB} = 2\hat{\boldsymbol{k}} \qquad \boldsymbol{\omega}_{AB} = \omega_{AB}\hat{\boldsymbol{k}}$$
$$\boldsymbol{\alpha}_{OB} = \boldsymbol{0} \qquad \boldsymbol{\alpha}_{AB} = \alpha_{AB}\hat{\boldsymbol{k}} \tag{4.43}$$

C. First, we use point O as a reference point to investigate point B.

$$\boldsymbol{v}_O = \boldsymbol{a}_O = \boldsymbol{0} \qquad \boldsymbol{r}_{B/O} = 1\hat{\boldsymbol{j}} \tag{4.44}$$

D. From the above information, we can find the velocity and acceleration of point B.

$$\boldsymbol{v}_B = \boldsymbol{v}_O + \boldsymbol{\omega}_{OB} \times \boldsymbol{r}_{B/O} = 2\hat{\boldsymbol{k}} \times 1\hat{\boldsymbol{j}} = -2\hat{\boldsymbol{\imath}} \tag{4.45}$$

$$\begin{aligned}\boldsymbol{a}_B &= \boldsymbol{a}_O + \boldsymbol{\alpha}_{OB} \times \boldsymbol{r}_{B/O} + \boldsymbol{\omega}_{OB} \times \left(\boldsymbol{\omega}_{OB} \times \boldsymbol{r}_{B/O}\right) \\ &= 2\hat{\boldsymbol{k}} \times \left(2\hat{\boldsymbol{k}} \times 1\hat{\boldsymbol{j}}\right) = 2\hat{\boldsymbol{k}} \times -2\hat{\boldsymbol{\imath}} \\ &= -4\hat{\boldsymbol{j}}\end{aligned} \tag{4.46}$$

C. Now, we can use point B as a reference point to investigate point A.

$$\boldsymbol{r}_{A/B} = 1\hat{\boldsymbol{\imath}} - 1\hat{\boldsymbol{\jmath}} \tag{4.47}$$

D. From the above information, we can find the velocity and acceleration of point A.

$$\begin{aligned} \boldsymbol{v}_A &= \boldsymbol{v}_B + \boldsymbol{\omega}_{AB} \times \boldsymbol{r}_{A/B} \\ &= -2\hat{\boldsymbol{\imath}} + \omega_{AB}\hat{\boldsymbol{k}} \times \left(1\hat{\boldsymbol{\imath}} - 1\hat{\boldsymbol{\jmath}}\right) \\ &= \left(\omega_{AB} - 2\right)\hat{\boldsymbol{\imath}} + \omega_{AB}\hat{\boldsymbol{\jmath}} \end{aligned} \tag{4.48}$$

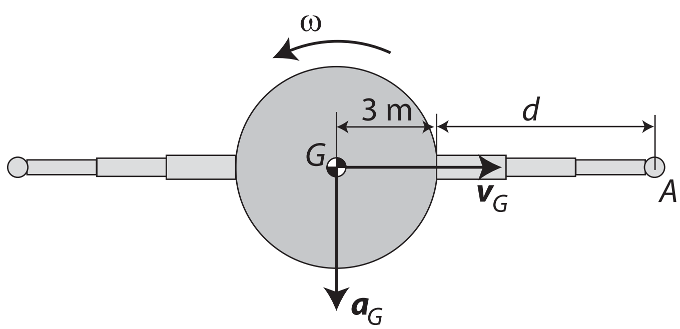
$$\begin{aligned} \boldsymbol{a}_A &= \boldsymbol{a}_B + \boldsymbol{\alpha}_{AB} \times \boldsymbol{r}_{A/B} + \boldsymbol{\omega}_{AB} \times \left(\boldsymbol{\omega}_{AB} \times \boldsymbol{r}_{A/B}\right) \\ &= -4\hat{\boldsymbol{\jmath}} + \alpha_{AB}\hat{\boldsymbol{k}} \times \left(1\hat{\boldsymbol{\imath}} - 1\hat{\boldsymbol{\jmath}}\right) + \omega_{AB}\hat{\boldsymbol{k}} \times \left(\omega_{AB}\hat{\boldsymbol{\imath}} + \omega_{AB}\hat{\boldsymbol{\jmath}}\right) \\ &= \left(\alpha_{AB} - \omega_{AB}^2\right)\hat{\boldsymbol{\imath}} + \left(\alpha_{AB} + \omega_{AB}^2 - 4\right)\hat{\boldsymbol{\jmath}} \end{aligned} \tag{4.49}$$

However, we know the velocity and acceleration of A must be in the vertical direction: $\boldsymbol{v}_A = v_A\hat{\boldsymbol{\jmath}}$ and $\boldsymbol{a}_A = a_A\hat{\boldsymbol{\jmath}}$.

$$\begin{aligned} \omega_{AB} - 2 &= 0 \\ \omega_{AB} &= v_A \\ \alpha_{AB} - \omega_{AB}^2 &= 0 \\ \alpha_{AB} + \omega_{AB}^2 - 4 &= a_A \end{aligned} \tag{4.50}$$

From these scalar equations, we find $\omega_{AB} = 2$ rad/s, $v_A = 2$ m/s, $\alpha_{AB} = 4$ rad/s^2, and $a_A = 4$ m/s^2.

Example 4.12
As the satellite moves through its orbit, its center of mass has a speed of 7740 m/s and an acceleration 9.01 m/s^2. The satellite is rotating at a rate of 0.5 rad/s and slowing down at 0.1 rad/s^2. Additionally, the satellite is extending a boom at a rate of 2 m/s and speeding up at 4 m/s^2. At the instant shown, the boom is extended a distance of $d = 7$ m. Calculate the velocity and acceleration of the end of the boom A.



A. Use coordinate vectors with $\hat{\boldsymbol{i}}$ pointing to the right, $\hat{\boldsymbol{j}}$ pointing up, and $\hat{\boldsymbol{k}}$ pointing out of the page.
B. Write the angular velocity and angular acceleration of the satellite.

$$\boldsymbol{\omega} = 0.5\hat{\boldsymbol{k}} \text{ rad/s} \qquad \boldsymbol{\alpha} = -0.1\hat{\boldsymbol{k}} \text{ rad/s}^2 \tag{4.75}$$

C. Use G as the reference point.

$$\boldsymbol{v}_G = 7740\hat{\boldsymbol{i}} \text{ m/s} \qquad \boldsymbol{a}_G = -9.01\hat{\boldsymbol{j}} \text{ m/s}^2 \qquad \boldsymbol{r}_{A/G} = 10\hat{\boldsymbol{i}} \text{ m} \tag{4.76}$$

D. The relative velocity and relative acceleration describe how A is moving relative to the satellite.

$$\boldsymbol{v}_{\text{rel}} = 2\hat{\boldsymbol{i}} \text{ m/s} \qquad \boldsymbol{a}_{\text{rel}} = 4\hat{\boldsymbol{i}} \text{ m/s}^2 \tag{4.77}$$

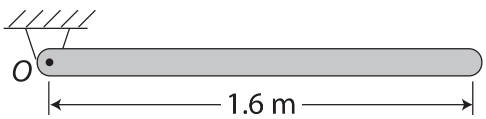
E. Evaluate the velocity and acceleration of A.

$$\begin{aligned} \boldsymbol{v}_A &= \boldsymbol{v}_G + \boldsymbol{\omega} \times \boldsymbol{r}_{A/G} + \boldsymbol{v}_{\text{rel}} \qquad (4.78) \\ &= 7740\hat{\boldsymbol{i}} + 0.5\hat{\boldsymbol{k}} \times 10\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{i}} \\ &= 7742\hat{\boldsymbol{i}} + 5\hat{\boldsymbol{j}} \text{ m/s} \end{aligned}$$

$$\begin{aligned} \boldsymbol{a}_A &= \boldsymbol{a}_G + \boldsymbol{\alpha} \times \boldsymbol{r}_{A/G} + \boldsymbol{\omega} \times \left(\boldsymbol{\omega} \times \boldsymbol{r}_{A/G}\right) + 2\boldsymbol{\omega} \times \boldsymbol{v}_{\text{rel}} + \boldsymbol{a}_{\text{rel}} \qquad (4.79) \\ &= -9.01\hat{\boldsymbol{j}} - 0.1\hat{\boldsymbol{k}} \times 10\hat{\boldsymbol{i}} + 0.5\hat{\boldsymbol{k}} \times 5\hat{\boldsymbol{j}} + 2\left(0.5\hat{\boldsymbol{k}}\right) \times 2\hat{\boldsymbol{i}} + 4\hat{\boldsymbol{i}} \\ &= 1.5\hat{\boldsymbol{i}} - 8.01\hat{\boldsymbol{j}} \text{ m/s}^2 \qquad (4.80) \end{aligned}$$

5.4 Example Problems

Example 5.4 (M&K 5ed 6/33)
The uniform 20 kg slender bar is pivoted at O and swings freely in the vertical plane. If the bar is released from rest in the horizontal position, calculate the initial values of the angular acceleration and the reaction forces at O.



1. Kinematics - Use rectangular coordinates with $\hat{\boldsymbol{\imath}}$ to the right, $\hat{\boldsymbol{\jmath}}$ up, and $\hat{\boldsymbol{k}}$ out of the page. As released from rest the angular velocity is zero, but we can expect the angular acceleration to be clockwise: $\boldsymbol{\omega} = \mathbf{0}$ and $\boldsymbol{\alpha} = -\alpha\hat{\boldsymbol{k}}$. The center of mass G is in the middle of the bar. We can use point O as a reference point to find the acceleration of G.

$$\boldsymbol{a}_O = \mathbf{0} \qquad\qquad \boldsymbol{r}_{G/O} = 0.8\hat{\boldsymbol{\imath}} \tag{5.25}$$

$$\begin{aligned}\boldsymbol{a}_G &= \boldsymbol{a}_O + \boldsymbol{\alpha}\times\boldsymbol{r}_{G/O} + \boldsymbol{\omega}\times\left(\boldsymbol{\omega}\times\boldsymbol{r}_{G/O}\right)\\ &= -\alpha\hat{\boldsymbol{k}}\times 0.8\hat{\boldsymbol{\imath}} = -0.8\alpha\hat{\boldsymbol{\jmath}}\end{aligned} \tag{5.26}$$

2. FBD - Drawing your own FBD, the weight mg acts downward through the center of mass, and let's guess the reaction forces R_x and R_y act to the right and up, respectively, at O.

$$\Sigma\boldsymbol{F} = R_x\hat{\boldsymbol{\imath}} + \left(R_y - mg\right)\hat{\boldsymbol{\jmath}} \tag{5.27}$$

We can sum moments about either G or O. We'll work it both ways, but let's start with G.

$$\Sigma\boldsymbol{M}_G = -0.8R_y\hat{\boldsymbol{k}} \tag{5.28}$$

3. EoM - Substitute the above results into the equations of motion.
$\Sigma\boldsymbol{F} = m\boldsymbol{a}_G$:

$$R_x\hat{\boldsymbol{\imath}} + \left(R_y - mg\right)\hat{\boldsymbol{\jmath}} = m\left(-0.8\alpha\hat{\boldsymbol{\jmath}}\right) \qquad \Rightarrow \qquad \begin{aligned}R_x &= 0\\ R_y - mg &= -0.8m\alpha\end{aligned} \tag{5.29}$$

$\Sigma\boldsymbol{M}_G = I_G\boldsymbol{\alpha}$:

$$-0.8R_y\hat{\boldsymbol{k}} = \frac{m1.6^2}{12}\left(-\alpha\hat{\boldsymbol{k}}\right) \qquad \Rightarrow \qquad -0.8R_y = -\frac{m1.6^2}{12}\alpha \tag{5.30}$$

Solving for the three equations of motion for the unknowns, $R_x = 0$, $R_y = 49.05$ N, and $\alpha = 9.20$ rad/s^2.

Now, let's go back and work the problem summing moment about O, just to show that we get the same answer.

2. FBD - Summing moments about O is actually convenient, because neither of the unknown reaction forces generate moments about O.

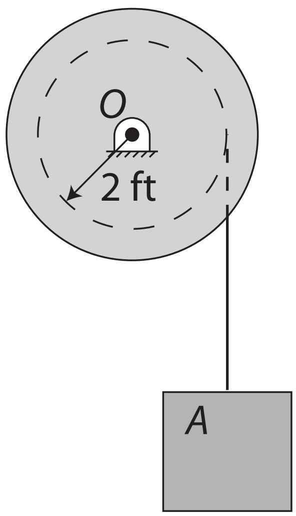
$$\Sigma \boldsymbol{M}_O = -0.8mg\hat{\boldsymbol{k}} \tag{5.31}$$

3. EoM - We write the new rotational equation of motion using the moment and mass moment of inertia about O.
$\Sigma \boldsymbol{M}_O = I_O\boldsymbol{\alpha}$:

$$-0.8mg\hat{\boldsymbol{k}} = \frac{m1.6^2}{3}\left(-\alpha\hat{\boldsymbol{k}}\right) \qquad \Rightarrow \qquad -0.8mg = -\frac{m1.6^2}{3}\alpha \tag{5.32}$$

Combining with Equation (5.29), gives an alternate set of three equations of motion. These questions are actually a little easier to solve: $R_x = 0$, $R_y = 49.05$ N, and $\alpha = 9.20$ rad/s^2.

Example 5.9
Block A is attached by a cable to a pulley whose center is the fixed axis O. The block has a weight of 30 lbs. The pulley has a weight of 50 lbs, an inner radius of 2 ft, and a radius of gyration of 1.75 ft. Find the acceleration of A, the angular acceleration of the pulley, and the tension in the cable, (a) assuming the pulley axis has zero friction, and (b) a 10 ft·lb moment due to friction acts at the pulley axis.



In particle kinetics, we looked at many problems involving pulleys. But in those previous problems, we implicitly assumed that the pulleys had negligible mass. In this problem, the dynamics of the large, massive pulley needs to be addressed, along with the block. Let's do part (a) first.
1. Kinematics - Use rectangular coordinates with $\hat{\boldsymbol{\imath}}$ to the right, $\hat{\boldsymbol{\jmath}}$ up, and $\hat{\boldsymbol{k}}$ out of the page. The angular acceleration of the pulley is going to be in the clockwise direction: $\boldsymbol{\alpha} = -\alpha\hat{\boldsymbol{k}}$. Point O is the center of mass of the pulley and has zero acceleration: $\boldsymbol{a}_O = \boldsymbol{0}$. We don't have to worry about any rotation of the block, but the acceleration is $\boldsymbol{a}_A = -2\alpha\hat{\boldsymbol{\jmath}}$.
2. FBD - We need to draw a FBD for each body. The pulley has the tension in each cable, reaction forces at O, and weight acting on it.

$$\Sigma \boldsymbol{F}_P = R_x\hat{\boldsymbol{\imath}} + (R_y - T - 50)\,\hat{\boldsymbol{\jmath}} \tag{5.56}$$

Since O is the center of mass of the pulley, it is our only choice to sum moments about.

$$\Sigma \boldsymbol{M}_O = -2T\hat{\boldsymbol{k}} \tag{5.57}$$

We also need to sum the forces acting on the block.

$$\Sigma \boldsymbol{F}_A = (T - 30)\,\hat{\boldsymbol{\jmath}} \tag{5.58}$$

3. EoM - Substituting into the equations of motion.
$\Sigma \boldsymbol{F}_P = m_P \boldsymbol{a}_O$:

$$R_x \hat{\boldsymbol{i}} + (R_y - T - 50)\,\hat{\boldsymbol{j}} = \boldsymbol{0} \qquad \Rightarrow \qquad \begin{matrix} R_x = 0 \\ R_y - T - 50 = 0 \end{matrix} \tag{5.59}$$

$\Sigma \boldsymbol{M}_O = I_O \boldsymbol{\alpha}$:

$$-2T\hat{\boldsymbol{k}} = \left(\frac{50}{32.2} \cdot 1.75^2\right)\left(-\alpha\hat{\boldsymbol{k}}\right) \qquad \Rightarrow \qquad 2T = \frac{153.125}{32.2}\alpha \tag{5.60}$$

$\Sigma \boldsymbol{F}_A = m_A \boldsymbol{a}_A$:

$$(T - 30)\,\hat{\boldsymbol{j}} = \frac{30}{32.2}\left(-2\alpha\hat{\boldsymbol{j}}\right) \qquad \Rightarrow \qquad T - 30 = -\frac{60}{32.2}\alpha \tag{5.61}$$

Solving these four equations, $\alpha = 7.07$ rad/sec^2 and $T = 16.8$ lb. This gives the acceleration of A as 14.2 ft/sec^2.

For part (b), the only change we need to make is to include the pure moment due to friction in $\Sigma \boldsymbol{M}_O$.
2. FBD - The friction moment acts in the counter clockwise direction, resisting the pulley's angular acceleration.

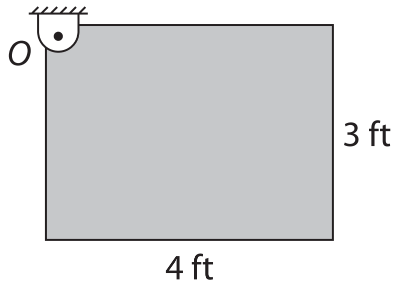
$$\Sigma \boldsymbol{M}_O = (-2T + 10)\,\hat{\boldsymbol{k}} \tag{5.62}$$

3. EoM - Only the rotational equation changes.
$\Sigma \boldsymbol{M}_O = I_O \boldsymbol{\alpha}$:

$$(-2T + 10)\,\hat{\boldsymbol{k}} = \left(\frac{50}{32.2} \cdot 1.75^2\right)\left(-\alpha\hat{\boldsymbol{k}}\right) \qquad \Rightarrow \qquad 2T - 10 = \frac{153.125}{32.2}\alpha \tag{5.63}$$

Solving the new set of equations gives $\alpha = 5.89$ rad/sec^2 and $T = 19.0$ lb, giving the acceleration of A as 11.8 ft/sec^2.

Example 6.6
The rectangular plate is hinged at corner O and rotates in a vertical plane. If the plate starts in the orientation shown with an angular velocity of 5 rad/sec, will it perform complete rotations about O? Or, oscillate like a pendulum?



Since O is a fixed axis, we can calculate the kinetic energy as $T_1 = \frac{1}{2}I_O\omega^2 = \frac{1}{2}(\frac{1}{3}m(3^2+4^2))5^2 = \frac{625}{6}m$. Defining point O as the zero height, the initial potential energy is $V_1 = mg(-1.5)$. As the plate swings, the nonpotential forces acting on it are the reaction forces at O. But since O is a fixed point, those forces do zero work. Let's investigate the configuration when the plate will have maximum potential energy. This potential energy will be $V_2 = mg\sqrt{1.5^2+2^2}$. The kinetic energy will be $T_2 = \frac{1}{2}(\frac{1}{3}m(3^2+4^2))\omega^2$.

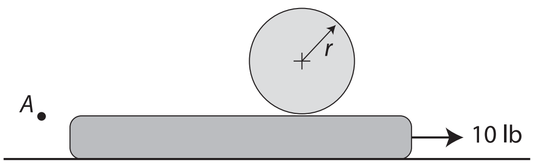
$$T_1 + V_1 + U_{NP,1\to 2} = T_2 + V_2$$

$$\frac{625}{6}m - m32.2\cdot 1.5 + 0 = \frac{25}{6}m\omega^2 + m32.2\sqrt{1.5^2+2^2} \tag{6.27}$$

$$\omega = \sqrt{-5.91} = 2.43i \tag{6.28}$$

The imaginary solution for the angular velocity means it's impossible for the plate to reach the orientation with its center of mass directly above O. Instead, it will oscillate like a pendulum.

Example 6.10
A platform is resting on a smooth surface, and a cylinder is resting on the platform. Both the platform and cylinder have weights of 50 lbs. A 10 lb force is then applied to the platform for 10 seconds. During this period the cylinder is observed to roll without slip on the platform. Find the speeds of the platform and the center of the cylinder at the end of the 10 second period.



Define $\hat{\boldsymbol{i}}$ pointing to the right, $\hat{\boldsymbol{j}}$ pointing up, and $\hat{\boldsymbol{k}}$ pointing out of the page. Consider the system consisting of the platform and cylinder. Initially, both start at rest, so the system has zero linear momentum. Over 10 seconds, the 10 pound force applies 100 lb·sec of impulse. At the end of the 10 seconds, the platform will have some velocity $v_P\hat{\boldsymbol{i}}$, and the center of the cylinder will have a velocity $v_C\hat{\boldsymbol{i}}$. Using this information, we can write the linear impulse-momentum principle.

$$\boldsymbol{p}_1 + \int_0^{10} \Sigma\boldsymbol{F}\mathrm{d}t = \boldsymbol{p}_2$$

$$0\hat{\boldsymbol{i}} + 100\hat{\boldsymbol{i}} = \frac{50}{32.2}v_P\hat{\boldsymbol{i}} + \frac{50}{32.2}v_C\hat{\boldsymbol{i}} \tag{6.51}$$

$$50\left(v_P + v_C\right) = 3220 \tag{6.52}$$

Next, let's look at the angular momentum of the cylinder by itself. Specifically, let's look at the angular momentum about the fixed reference point A labeled in the diagram. Initially, the cylinder has zero angular momentum about A. During the 10 second period, the forces acting on the cylinder are weight due to gravity and normal & friction forces exerted by the platform. The angular impulses about A due to the weight and normal forces cancel each other out. The friction force exerts zero moment about A and therefore zero angular impulse about A. At the end of the end of the 10 seconds, the cylinder's angular momentum about A will be related to the moment of its linear momentum and its angular velocity $\boldsymbol{\omega} = \omega\hat{\boldsymbol{k}}$. Since the cylinder is rolling without slipping on the moving platform, the velocity of its center and its angular velocity are linked.

$$\boldsymbol{v}_C = \boldsymbol{v}_P + \boldsymbol{\omega}\times\boldsymbol{r}_{C/P} \tag{6.53}$$

$$v_C\hat{\boldsymbol{i}} = v_P\hat{\boldsymbol{i}} + \omega\hat{\boldsymbol{k}}\times r\hat{\boldsymbol{j}} = \left(v_P - \omega r\right)\hat{\boldsymbol{i}}$$

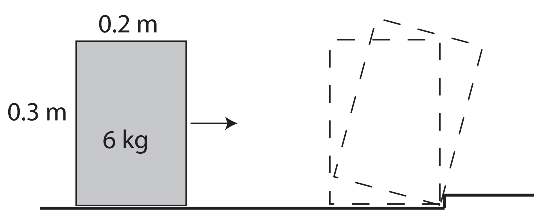
$$\omega = \frac{v_P - v_C}{r} \tag{6.54}$$

$$\begin{aligned}
\boldsymbol{h}_{A,1} + \int_0^{10} \Sigma \boldsymbol{M}_A \mathrm{d}t &= \boldsymbol{h}_{A,2} \\
0\hat{\boldsymbol{k}} + 0\hat{\boldsymbol{k}} &= -\frac{50}{32.2} v_c r \hat{\boldsymbol{k}} + \frac{1}{2} \cdot \frac{50}{32.2} r^2 \omega \hat{\boldsymbol{k}} \qquad (6.55) \\
0 &= -\frac{50}{32.2} v_c r + \frac{25}{32.2} r \left(v_P - v_C\right) \\
25 v_P - 75 v_C &= 0 \qquad (6.56)
\end{aligned}$$

Solving these two equations gives $v_c = 16.1$ ft/sec and $v_p = 48.3$ ft/sec. We were able to find these solutions without ever calculating the friction force between the platform and cylinder.

Example 6.11

A rectangular block is sliding across a smooth surface with a speed of 1.2 m/s. It hits a small curb on the surface, causing it to rotate about the lower right corner. (Neglect the height of the curb.) Find the angular velocity of the block immediately after it hits the curb.



Label the curb as point C, and define $\hat{\boldsymbol{k}}$ pointing out of the page. The initial angular momentum of the block about C is $\boldsymbol{h}_C = -6 \cdot 1.2 \cdot 0.15\hat{\boldsymbol{k}} = -1.08\hat{\boldsymbol{k}}$ N·m·s. During the impact reaction forces at C act on the block. But they generate zero moment about C, and therefore zero angular impulse about C. After the impact, the block is rotating about C.

Using this information, we can write the angular impulse-momentum principle about C.

$$\begin{aligned}
\boldsymbol{h}_{C,1} &= \boldsymbol{h}_{C,2} \\
-1.08\hat{\boldsymbol{k}} &= \left(\frac{1}{3} 6 \left(0.2^2 + 0.3^2\right)\right) \left(-\omega \hat{\boldsymbol{k}}\right) \qquad (6.57) \\
\omega &= 4.15 \text{ rad/sec}
\end{aligned}$$

Formulas

$$v = v_0 + at$$

$$r = r_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^{\;2} + 2a(s - s_0)$$

$$\boldsymbol{r} = x\hat{\boldsymbol{i}} + y\hat{\boldsymbol{j}} = r\hat{\boldsymbol{e}}_r$$

$$\boldsymbol{v} = \dot{x}\hat{\boldsymbol{i}} + \dot{y}\hat{\boldsymbol{j}} = \dot{r}\hat{\boldsymbol{e}}_r + r\dot{\theta}\hat{\boldsymbol{e}}_\theta = v\hat{\boldsymbol{e}}_t$$

$$\begin{aligned}\boldsymbol{a} &= \ddot{x}\hat{\boldsymbol{i}} + \ddot{y}\hat{\boldsymbol{j}} \\ &= \left(\ddot{r} - r\dot{\theta}^2\right)\hat{\boldsymbol{e}}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\boldsymbol{e}}_\theta \\ &= \dot{v}\hat{\boldsymbol{e}}_t + \frac{v^2}{\rho}\hat{\boldsymbol{e}}_n\end{aligned}$$

$$\boldsymbol{r}_A = \boldsymbol{r}_B + \boldsymbol{r}_{A/B}$$

$$\boldsymbol{v}_A = \boldsymbol{v}_B + \boldsymbol{v}_{A/B}$$

$$\boldsymbol{a}_A = \boldsymbol{a}_B + \boldsymbol{a}_{A/B}$$

$$\boldsymbol{v}_A = \boldsymbol{v}_B + \boldsymbol{\omega} \times \boldsymbol{r}_{A/B} + \boldsymbol{v}_{rel}$$

$$\boldsymbol{a}_A = \boldsymbol{a}_B + \boldsymbol{\alpha} \times \boldsymbol{r}_{A/B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}_{A/B}) + 2\boldsymbol{\omega} \times \boldsymbol{v}_{rel} + \boldsymbol{a}_{rel}$$

$$g = 32.2\ \text{ft/sec}^2 = 9.81\text{m/s}^2$$

$$f_s \le \mu_s N$$

$$f_k = \mu_k N$$

$$I_C = I_G + md^2$$

$$\mathrm{I}_C = mk_c^2$$

$$\Sigma \boldsymbol{F} = m\boldsymbol{a}_G$$

$$\Sigma \mathbf{M}_C = \mathrm{I}_C \boldsymbol{\alpha}$$

$$U_{1\to 2} = \int_1^2 \boldsymbol{F} \bullet d\boldsymbol{r}$$

$$T_{\text{particle}} = \frac{1}{2}mv^2$$

$$T_{\text{body}} = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2 = \frac{1}{2}I_O\omega^2$$

$$T_1 + V_1 + U_{\text{np},1\to 2} = T_2 + V_2$$

$$\boldsymbol{p}_{\text{particle}} = m\boldsymbol{v}$$

$$\boldsymbol{p}_{\text{body}} = m\boldsymbol{v}_G$$

$$\int_1^2 \Sigma \mathbf{F} \mathrm{d}t = \mathbf{p}_2 - \mathbf{p}_1$$

$$\mathbf{h}_{C,\text{particle}} = \mathbf{r} \times m\mathbf{v}$$

$$\mathbf{h}_{C,\text{body}} = \mathbf{r}_{G/C} \times m\mathbf{v}_G + I_G\boldsymbol{\omega}$$

$$\mathbf{h}_{G,\text{body}} = I_G\boldsymbol{\omega}$$

$$\mathbf{h}_{O,\text{body}} = I_O\boldsymbol{\omega}$$

$$\int_1^2 \Sigma \boldsymbol{M}_C\ \mathrm{d}t = \boldsymbol{h}_{C,2} - \boldsymbol{h}_{C,1}$$